HETEROGENEOUS EXPECTATIONS AND WEALTH INEQUALITY*

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Abstract

Using microdata, we document substantial heterogeneity in households' stock return expectations that persists over time and correlates strongly with wealth. We develop a rich heterogeneous agent model where this belief dispersion arises endogenously through learning from experience, creating a feedback loop between expectations and portfolio choices. The model matches key features of the joint distribution of expectations, portfolio returns, and wealth in the data. Belief heterogeneity amplifies wealth concentration through two channels: optimistic households both save more and choose riskier portfolios, generating higher realized returns that further reinforce their optimistic beliefs. Relative to a homogeneous-beliefs benchmark, heterogeneous expectations increase the wealth share of the top 1% by 50%. Methodologically, we show that Internal Rationality -where households learn about prices directly rather than needing to forecast entire distributions - makes heterogeneous agent models with aggregate risk both more realistic and more tractable.

Keywords: Heterogeneous Expectations, Wealth Inequality, Internal Rationality.

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1. Introduction

The rise in wealth inequality stands as one of the most pressing economic challenges of our time. Over the past decades, wealth has become increasingly concentrated at the top of the distribution across advanced economies. In the United States, for instance, the wealth share of the top 1% has risen from 23% in 1980 to over 35% today.[1] This trend has profound implications for economic and political stability as well as for the conduct of fiscal and monetary policy, making it imperative to understand the mechanisms driving wealth concentration.

Partly responding to that challenge, modern macroeconomics has made considerable progress in incorporating heterogeneity into its analytical framework. Contemporary models routinely feature rich heterogeneity in dimensions such as income processes, saving rates, portfolio choices, or entrepreneurial ability. However, a striking exception remains: the treatment of expectations. The majority of models impose what Sargent termed a "Communism of Beliefs" - the assumption that all agents share identical expectations about future economic outcomes.[2]

Yet, do economic agents actually hold similar beliefs? Survey data suggests otherwise. Rather than clustering around a consensus forecast, household expectations display a rich, full distribution. Figure 1 illustrates this point with the cross-section distribution of expected capital gains in the stock market. This striking heterogeneity in beliefs motivates our investigation of four key questions about the relationship between expectations and wealth inequality.

Our first question is: What are the key facts of the distribution of survey expected returns? Using a unique public dataset, the RAND - American Life Panel survey, that combines detailed portfolio information with elicited beliefs, we document two robust patterns. First, in line with the findings of Giglio et al. (2021) using a panel of Vanguard investors, we find substantial and persistent disagreement in expected returns: instead of jumping between optimism and pessimism, optimists (pessimists) tend to remain optimists (pessimists). Second, differently from Giglio et al. (2021)'s data, we have a representative sample of the US population. That allows us to uncover a strong positive correlation between wealth and return expectations, with wealthier households systematically reporting higher expected returns and smaller

[1] Data from `realtimeinequality.org/`

[2] Recent work by Tobias et al. (2022) represents a notable exception, incorporating heterogeneous beliefs about inflation, unemployment, and housing prices. However, their retention of the Rational Expectations assumption leads to significant computational complexity. Moreover, their framework does not address beliefs about risky returns - a crucial determinant of top wealth shares.

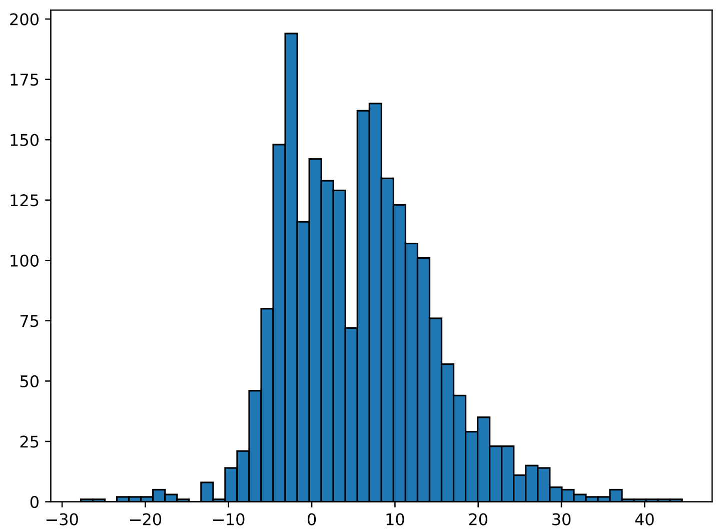

Figure 1: Cross-section of expected returns. 2015 - RAND dataset.

forecast errors.[3]

The second question is: What is a reasonable way of modeling Heterogeneous Expectations? We propose a framework based on the concept of Internal Rationality developed by Adam and Marcet (2011), where agents make optimal decisions given their subjective beliefs but do not possess full knowledge about market characteristics. Under this approach, agents treat prices as a non-degenerate stochastic process and form beliefs by filtering the price sequence. Thus, under imperfect knowledge, agents' subjective model of prices becomes a primitive of the model, necessary to complete the probability measure they use for optimization. We conjecture a state-space model of prices with different layers of heterogeneity. In particular, we include heterogeneous long-run views about the fundamental value of the asset that can replicate the statistical individual fixed effects found in the microdata. Thus, we retain rationality while include flexibility that allows to match the data.

Our third question tackles a key technical challenge: Does incorporating heterogeneous expectations make heterogeneous agent models computationally even less tractable? We argue that our framework actually simplifies the solution algorithm. Under Internal Rationality, prices become state variables that are directly forecasted by the agents using their subjective models. For this reason, there is no need for agents to forecast entire distributions of wealth and income in order to forecast prices. This approach avoids Moll (2024)'s "nonsensical problem" critique, keeping the theoretical rigor while simplifying the computational burden. The key insight

[3]Optimism is also associated to higher financial literacy, and lower risk aversion.

is that by allowing households to form expectations directly about prices rather than forcing them to forecast prices through distributions forecasts, we avoid the infinite-dimensional state space problem.[4] Thus, our framework matches microdata heterogeneity while increasing computational tractability.

The fourth question is: How much do Heterogeneous Expectations influence Wealth Inequality? Through numerical analysis of a calibrated model that matches the joint distribution of income and expectations and generate wealth and returns distributions qualitatively in line with the data, we show that belief heterogeneity substantially amplifies wealth concentration through two channels. First, differences in expected returns generate systematic variations in savings rates across the wealth distribution. Second, heterogeneous expectations lead to persistent differences in portfolio composition, with more optimistic (and typically wealthier) households maintaining higher equity exposures. These mechanisms increase the wealth share of the top 10% by approximately 20% and that of the top 1% by 50%, relative to a model with homogeneous expectations.

While our baseline model treats heterogeneous long-run views as persistent statistical fixed effects, we extend the analysis to examine their origins. Building on a vast literature documenting learning from experience (e.g., Malmendier and Nagel (2016)), we show that fixed effects in expectations emerge naturally when agents learn from their own experienced portfolio returns. This generates a powerful feedback mechanism: investors who experience higher returns become more optimistic, allocate more wealth to risky assets, and earn higher average returns, further reinforcing their optimistic beliefs. This feedback loop between expected and realized returns creates an additional amplification channel for wealth inequality, as successful investors become increasingly optimistic and wealthy over time. This extension provides a natural microfoundation for the persistent heterogeneity in beliefs we observe in the data while highlighting a novel mechanism through which initial differences in investment outcomes can compound into substantial wealth disparities.

Our analysis contributes to several active research areas in macroeconomics and finance. First, we extend the literature on measured expectations, joining works by Greenwood and Shleifer (2014), Coibion and Gorodnichenko (2015), and Kohlhas and Walther (2021) in documenting systematic patterns in belief formation. Our finding of a strong association between wealth and expectations complements the

[4] Moll (2024) argues that standard approaches to heterogeneous agent models with aggregate risk lead to a "nonsensical problem" where households must forecast infinite-dimensional objects to forecast prices and calls for alternative approaches. Our framework provides such alternative.

results of Giglio et al. (2021) on the persistence of disagreement, showing that when looking at the whole population, wealth is indeed very related with optimism.

This paper also advances our understanding of wealth inequality mechanisms. Previous research has identified various drivers of wealth concentration, including bequests (e.g., De Nardi (2004)), preference heterogeneity (e.g., Krusell and Smith (1998)), earnings risk (e.g., De Nardi, Fella, and Pardo (2016)), return heterogeneity (e.g., Fagereng et al. (2020)), entrepreneurship (e.g., Quadrini (2000)), and tax policy (e.g., Hubmer, Krusell, and Smith Jr (2021)). We contribute to this literature by providing a novel microfoundation for heterogeneous returns and portfolio choices, showing how differences in beliefs can generate and amplify wealth inequality. Moreover, our methodological contribution - combining Internal Rationality with the Parameterized Expectations Algorithm- provides a tractable framework for analyzing heterogeneous agent models with aggregate risk and endogenous portfolio, opening new possibilities for future research in this area, which contrasts with other much more complex approaches proposed for similar models with Rational Expectations (e.g., Fernández-Villaverde and Levintal (2024)).

2. Facts about the distribution of expectations

Our analysis uses data from the RAND American Life Panel (ALP), a regular household survey conducted between 2008 and 2017. The ALP is uniquely suited for studying the relationship between wealth and expectations, as it is the only public panel dataset that simultaneously tracks both household wealth and return expectations. The unbalanced panel includes approximately 3,000 participants, providing a comprehensive view of household belief formation over time.

To elicit return expectations, the survey asks participants three questions about their beliefs regarding future returns of blue-chip stocks over a one-year horizon. Specifically, respondents provide probability assessments for: (1) positive returns of any magnitude, (2) returns exceeding 20 percent, and (3) losses exceeding 20 percent.[5] From these responses, we can construct implied probabilities for different return ranges. For instance, a typical response pattern of 65%, 15%, and 10% for the three questions respectively implies a 50% probability of returns between 0%

[5]The exact framing of one question is (the others are similar): *By next year at this time, what are the chances that mutual fund shares invested in blue-chip stocks like those in the Dow Jones Industrial Average will have increased in value by more than 20 percent compared to what they are worth today?*

and 20%, and a 25% probability of returns between -20% and 0%. We exclude observations that violate internal consistency (i.e., yield negative probabilities).

To ensure sufficient time-series variation for our analysis of belief persistence, we restrict our sample to participants with at least 10 successive quarterly observations. This filtering reduces our sample to 2,321 households, though our results are robust to alternative minimum observation thresholds. Table 1 illustrates the sample size under different minimum observation requirements, ranging from 3,027 households with at least three observations to 303 households with fifty or more observations.

Minimum Observations	3	4	5	6	10	30	50
Number of Households	3,027	2,969	2,882	2,764	2,321	785	303

Table 1: *Sample Size by Minimum Number of Successive Observations. This table shows the number of households in our sample that meet different minimum observation requirements. Data from RAND American Life Panel, 2008-2017.*

Using data from the RAND American Life Panel, we document that household expectations exhibit systematic deviations from rational expectations, consistent with patterns documented in other datasets. We focus on two key features: forecast error predictability and perpetual disagreement.

Forecast Error Predictability

We find robust evidence of both extrapolation and overreaction in household expectations. Following Kohlhas and Walther (2021), we estimate:

$$R_{t+1} - \beta_t^i = \alpha^i + bR_t + \varepsilon_t^i \tag{1}$$

where $\beta_t^i \equiv \mathbb{E}_t^{\mathcal{P}^i}(R_{t+1})$ represents household i's expectation at time t. To test for overreaction, we estimate:

$$R_{t+1} - \beta_t^i = \alpha^i + d(\beta_t^i - \beta_{t-1}^i) + \varepsilon_t^i \tag{2}$$

The negative coefficient $d = -0.37$ suggests that households overreact to new information, consistent with recent evidence from Bordalo et al. (2020).

Persistent Heterogeneity in Beliefs

A striking feature of our data is the persistence of disagreement among households. To quantify this, we estimate:

$$\beta_t^i = \alpha^i + \mu_t + v_t^i \tag{3}$$

where α^i captures individual fixed effects and μ_t represents time fixed effects. Individual fixed effects alone explain 50% of the variation in expectations, while time effects account for only 1%. Adding both fixed effects increases the R^2 marginally to 52%. This dominance of individual fixed effects, illustrated in Figure ??, aligns with findings by Giglio et al. (2021) using Vanguard data.

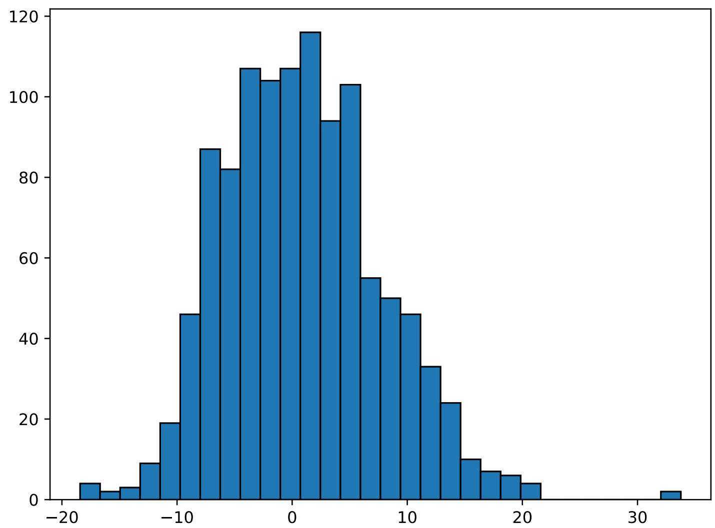

Figure 2: Distribution of individual fixed effects. *This figure shows the distribution of individual fixed effects in expected returns α^i. Data from RAND American Life Panel.*

These empirical patterns - forecast error predictability and persistent heterogeneity - inform our modeling choices in Section 3, where we develop a framework that can accommodate both features while maintaining tractability.

The joint distribution of expectations and wealth

Our analysis of the RAND American Life Panel reveals striking patterns in the relationship between household wealth, and return expectations. Expected returns exhibit strong positive correlations with economic variables across the wealth distribution, as shown in Table 2 and Figure 3.

	Top 1	Top 10	Middle 40	Bottom 50
$\mathbb{E}_t^{S^i}(R_{t+1})$	8.1%	7.7%	6.1%	4.3%
FE_{t+1}	0.63	1.69	2.44	3.76

Table 2: Expected Returns and Forecast Errors by Wealth Group. *This table reports average expected returns and forecast errors by wealth group. Forecast errors are measured in percentage points. Data from RAND American Life Panel, 2008-2017.*

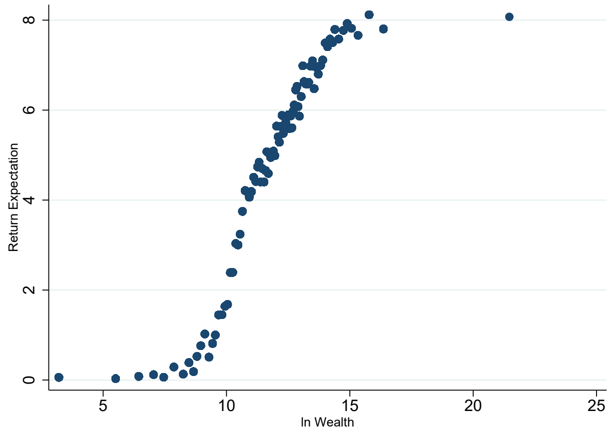

Figure 3: Correlation between wealth and expected returns. RAND dataset.

This pattern is particularly pronounced at the top of the wealth distribution. As Table 2 shows, households in the top 1% expect substantially higher returns than the rest of the population. Moreover, these wealthy households demonstrate superior forecasting ability, with forecast errors less than one-fifth those of the bottom 50%.

The data also reveal systematic differences in risk attitudes across the wealth distribution. Table 3 shows that wealthy households are substantially more likely to report willingness to take financial risks for higher returns. Among the top 1%, over a quarter report willingness to take substantial financial risks, compared to less than 9% in the bottom decile.

These patterns in risk attitudes coincide with substantial heterogeneity in financial sophistication. Table 4 demonstrates that both self-assessed financial knowledge and risk-taking capacity increase monotonically with wealth. The joint distribution of expectations, risk attitudes, and financial sophistication suggests that wealth accumulation may be intimately connected to both the willingness and the ability to form accurate return expectations and take calculated investment risks.

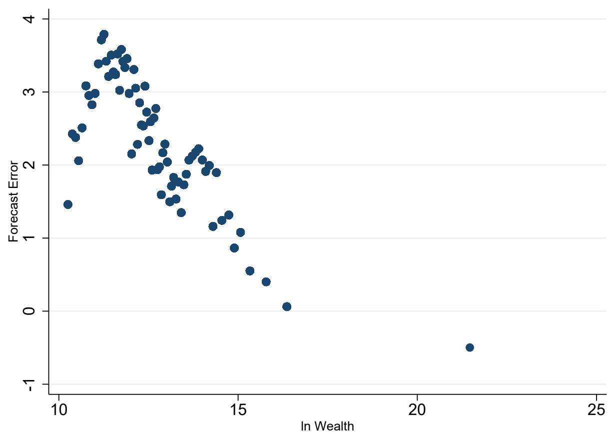

Figure 4: Correlation between wealth and forecast errors. RAND dataset.

Percentile	10	50	90	95	99
Substantial risk	8.5%	4.8%	3.5%	8.31%	25.96%
Above average risk	11%	10.8%	41.8%	45.4%	32.7%
Average risk	28.6%	42.68%	43.7%	44.1%	36.5%
No risk	51.83%	41.72%	10.93%	2.24%	4.8%

Table 3: Risk Attitudes by Wealth Percentile. *This table reports the distribution of risk attitudes across wealth percentiles. Categories represent responses to: "Which of the following statements comes closest to describing the amount of financial risk that you are willing to take when you save or make investments?" Data from RAND American Life Panel, 2008-2017.*

3. A HA model with heterogeneous expectations

It is a model with both idiosyncratic and aggregate income shocks, incomplete markets (as in Krusell and Smith (1998) but without capital), endogenous portfolio choice, Internal Rationality and heterogeneous expectations.

Household problem. Consider an endowment economy populated by I groups of agents, $i \in [1, I]$, who face a savings-consumption and a portfolio choice problem:

$$\max_{\{C_t^i, S_t^i, B_t^i\}_{t=0}^{\infty}} \mathbb{E}_0^{\mathcal{P}^i} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \tag{4}$$

subject to the budget constraint

$$C_t^i + P_t S_t^i + B_t^i \leq W_t^i + (D_t + P_t) S_{t-1}^i + R B_{t-1}^i \tag{5}$$

Wealth Percentile	10	50	90	95	99
Financial Knowledge	6.8	7.4	8.7	8.6	8.9
Risk-taking	3.1	4.2	5.9	7.0	7.5

Table 4: Financial Sophistication by Wealth Percentile. *This table reports self-assessed financial knowledge and risk-taking capacity on a scale of 1 to 10 across wealth percentiles. Data from RAND American Life Panel, 2008-2017.*

and asset holding limits, including borrowing constraints

$$\underline{S} \leq S_t^i \leq \bar{S} \tag{6}$$

$$\underline{B} \leq B_t^i \leq \bar{B} \tag{7}$$

given initial endowments $S_0^i = 1/I$ and $B_0^i = 0$.

Income processes. This is a small open endowment economy. Aggregate wages are defined in terms of the wage-dividend ratio, with i.i.d. growth around a mean $1 + \overline{WD}$

$$ln\left(1 + \frac{W_t}{D_t}\right) = (1-p)ln(1 + \overline{WD}) + p\ ln\left(1 + \frac{W_{t-1}}{D_{t-1}}\right) + ln\varepsilon_t^W \tag{8}$$

Dividends growth is i.i.d., fluctuating around an average growth rate a

$$lnD_t = lnD_{t-1} + a + +ln\varepsilon_t^D \tag{9}$$

Aggregate shocks are correlated reflecting underlying macroeconomic factors that affect the different sources of aggregate income. They follow a multinormal distribution with covariance σ_{DW}

$$\begin{pmatrix} ln\varepsilon_t^D \\ ln\varepsilon_t^W \end{pmatrix} \sim \mathcal{N}\left(-\frac{1}{2}\begin{pmatrix} \sigma_D^2 \\ \sigma_W^2 \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DW} \\ \sigma_{DW} & \sigma_W^2 \end{pmatrix}\right), \tag{10}$$

People can borrow goods at an exogenous rate R, subject to their borrowing constraints. At rate R, international investors are always willing to lend/borrow from domestic households, as a counterpart for goods imports/exports.

Individual wages are exogenous, expressed as a random share $\tilde{w}$ of aggregate ones and affected by an idiosyncratic income shock $\nu_t^i \sim \mathcal{N}(0, \sigma_\nu^2)$

$$\frac{W_t^i}{D_t} = \tilde{w}^i \frac{W_t}{D_t} \nu_t^i \tag{11}$$

We require $\sum_i \tilde{w}^i = 1$. To that end, let

$$\tilde{w}^i = \frac{w^i}{\sum_j w^j} \tag{12}$$

w^i is simulated from a Mixed Lognormal-Pareto distribution such that

$$w^i \sim \begin{cases} log\mathcal{N}(\mu, \sigma_w^2) & \text{if } i < \underline{w} \\ Pareto(\underline{w}, \alpha) & \text{if } i \geq \underline{w} \end{cases} \tag{13}$$

This mixture captures in a parsimonious way the approximately log-Normal shape of the real-world income distribution, while correcting for the particularly right heavy tail with a Pareto distribution.

Agents' Belief System. Agents are endowed with perfect knowledge of the law of motions for dividends and wages given by equations (8)-(9)-(10)-(11)-(12). However, agents have only imperfect knowledge about price formation. Thus, the underlying probability space $(\boldsymbol{\Omega}, \mathcal{B}, \mathcal{P}^i)$ with a typical element $\omega \in \boldsymbol{\Omega}$ with $\omega = \{D_t, W_t^i, P_t\}_{t=0}^{\infty}$. In our model, agents treat prices as an exogenous stochastic process they must forecast. This imperfect knowledge requires specifying agents' subjective model of prices to complete the probability measure they use for optimization.

We conjecture a state-space model of prices with different layers of heterogeneity. Investors from the sentiment group i possess the following subjective model about stock prices

$$\begin{aligned} lnP_t &= lnP_{t-1} + lnb_t^i + ln\varepsilon_t^{P,i} \\ lnb_t^i &= (1-\rho^i)ln\bar{\beta}^i + \rho_i lnb_{t-1}^i + ln\zeta_t^i \\ ln\varepsilon_t^{P,i} &\sim i.i.\mathcal{N}(-\frac{\sigma_P^2}{2}, \sigma_P^2),\ ln\zeta_t^i \sim i.i.\mathcal{N}(-\frac{\sigma_\zeta^2}{2}, \sigma_\zeta^2) \end{aligned} \tag{14}$$

where b_t^i represents the permanent price growth component, $\varepsilon_t^{P,i}$ a transitory innovation to prices and ζ_t^i an innovation to the permanent component of returns.[6] The

[6] The noisy price component is comprised of two independent zero-mean normal components

$$\varepsilon_t^{R,i} = \varepsilon_{t+1}^{R1,i} + \varepsilon_t^{R2,i} \tag{15}$$

permanent component, b_t^i, follows an auto-regressive process with persistence ρ^i and mean $\bar{\beta}^i$. The latter represents the perceived long-term return of sentiment group i and captures the stylized fact that we document in section 2 related to the presence of fixed effects in the cross section of the distribution of survey expectations. The permanent component of price growth, b_t, is not observed and is optimally estimated using the information available from the price signals. Given their belief system from equation 14, the optimal posterior distribution of the permanent component of prices is

$$lnb_t^i \sim \mathcal{N}(ln\beta_t^i, (\sigma^i)^2) \tag{16}$$

where $(\sigma^i)^2$ is the steady state variance of the posterior given by

$$(\sigma^i)^2 = \frac{(\sigma_\zeta^i)^2 + \sqrt{((\sigma_\zeta^i)^4 + 4(\sigma_\zeta^i)^2\sigma_R^2}}{2} \tag{17}$$

β_t^i is the conditional mean, which evolves according to the Kalman updating equation

$$ln\beta_t^i = (1-\rho^i)(1-g^i)ln\bar{\beta}^i + \rho^i ln\beta_{t-1}^i + g^i\left(ln\frac{P_{t-1}}{P_{t-2}} - \rho^i ln\beta_{t-1}^i\right) + g^i\varepsilon_t^{P1,i} \tag{18}$$

where $g^i = \frac{(\sigma^i)^2}{(\sigma^i)^2+\sigma_R^2}$ represents the steady state Kalman gain, entailing different views on the signal-to-noise ratio of the price signals. The shock $\varepsilon_t^{P1,i}$ is an information shock to the beliefs of agents from group i. Altogether,

$$\mathbb{E}_t^{\mathcal{P}^i}\left[\frac{P_{t+1}}{P_t}\right] = \kappa^i\bar{\beta}^{i(1-\rho^i)}(\beta_t^i)^{\rho i} \tag{19}$$

Qualitatively, equation (18) contains elements that might replicate the key observations from surveys: the heterogeneous long-run views about the fundamental value of the asset can be linked to the individual fixed-effects and the perpetual disagreement; the different views about the signal-to-noise ratio of the price signals can lead to different degrees of extrapolation; the persistence parameter can be directly linked to the persistence from the survey; the fact that all agents use the same price information would generate a high comovement between sentiment groups.

Based on the empirical evidence of Section 2, we let $\rho^i = \rho\ \forall i$. There are three parameters that capture the heterogeneity of agents in terms of income and expectations. Let $\boldsymbol{g} = [\underline{g}, \bar{g}]$, $\bar{\boldsymbol{\beta}} = [\underline{\bar{\beta}}, \bar{\bar{\beta}}]$ be the support for the learning gain, the long-

and we assume as in Adam (2017) that only $ln\varepsilon_t^{R1,i}$ is observed at time t, giving rise to the lag-updating equation that is usually found in the learning literature.

term price growth. The capture the comovement between $(w, g, \bar{\beta})$ documented in Section 2, we proceed as follows. First, we standardize the stationary income shares w^i. Then, we generate noise components at the individual level z^i and related to $\bar{\beta}$, $\varepsilon_{\bar{\beta}}$ and to g, ε_g. These noise is i.i.d. standard normally distributed. To introduce a correlation, we simulate

$$\tilde{\beta}^i = \rho_{\bar{\beta},w} w_z^i + \rho_{g,w} z^i + (1 - \rho_{\bar{\beta},w}^2 - \rho_{g,w}^2)^{0.5} \varepsilon_{\bar{\beta}}^i \tag{20}$$

$$\tilde{g}^i = \rho_{\bar{\beta},w} w_z^i + \rho_{g,w} z^i + (1 - \rho_{\bar{\beta},w}^2 - \rho_{g,w}^2)^{0.5} \varepsilon_g^i \tag{21}$$

where w_z^i is the standardized income share and ρ_{xy} represents the correlation between x and y. Finally, $\tilde{\beta}^i$ and $\tilde{g}^i$ are min-max normalized to remain within the support $\boldsymbol{g}$ and $\bar{\boldsymbol{\beta}}$.

3.1. Competitive Equilibrium

Sequential Competitive Equilibrium. Given the exogenous processes, agents' probability measures initial wealth holdings and a given the international interest rate R, a sequential competitive equilibrium is a set of stochastic sequences for all quantities $\{C_t^i, B_t^i, S_t^i\}_{it}$ and prices $\{P_t\}_t$ such that the following conditions hold:

1. Optimal intertemporal condition of households.
2. Non-arbitrage condition between bonds and stocks.
3. Market clearing conditions for the
   (a) Goods market: $\sum_{i=1}^I C_t^i = D_t + W_t - B_t + RB_{t-1}$ where $B_t = \sum_{i=1}^I B_t^i$
   (b) Stock market: $\sum_{i=1}^I S_t^i = 1$

Recursive Formulation. To recast the problem in recursive form, we need to specify the aggregate and individual state variables. Recall the budget constraint:

$$C_t^i + P_t S_t^i + B_t^i \leq W_t^i + (D_t + P_t) S_{t-1}^i + RB_{t-1}^i \tag{22}$$

In each period, household resources depend on the chosen portfolio (S_{t-1}^i, B_{t-1}^i), its wages W_t^i, the realization of dividends which depends on the aggregate shock ε_t^D and stock prices P_t. Then, a natural way to summarize household i position at the beginning of each period is by all these variables, which involves both individual and aggregate states $\boldsymbol{x}_t^i = (S_{t-1}^i, B_{t-1}^i, \nu_t^i, \varepsilon_t^W, \varepsilon_t^D, P_t)$.

Thus, household's problem can be recast as (without superscript i to save notation):

$$V(\boldsymbol{x}) = \max_{c,s',b'} u(c) + \delta \mathbb{E}^{\mathcal{P}}[V(\boldsymbol{x}')] \tag{23}$$

subject to the budget constraint and the asset holdings limits.

States involve past individual choices, exogenous variables and prices, which are an endogenous aggregate variable. In equilibrium, there will be a pricing function, which maps resources to prices. Let $h^s(x_t^i)$ be the policy function for stocks. Equilibrium prices are the ones clearing the stock market,

$$\sum_i h^s(S_{t-1}^i, B_{t-1}^i, \nu_t^i, \varepsilon_t^W, \varepsilon_t^D, P_t) = 1 \tag{24}$$

That equation can be solved for P_t such that the equilibrium pricing function p depends on the distribution of asset holdings, wages and the aggregate shocks, that is,

$$P_t = p(\bar{\Gamma}_t, \varepsilon_t^W, \varepsilon_t^D) \tag{25}$$

where $\bar{\Gamma}_t$ is the joint cumulative distribution function of asset holdings and wages, measuring how many households are below particular combinations of stock, bond holdings and idiosyncratic wage shocks, that is,

$$\bar{\Gamma}_t \equiv \bar{\Gamma}(s, b, \nu) = \sum_{i=1}^{I} \mathbf{1}\Big\{ S_{t-1}^i \leq s, B_{t-1}^i \leq b, \nu_t^i \leq \nu \Big\} \tag{26}$$

where $\mathbf{1}(\cdot)$ is the indicator function.

In models of Rational Expectations, it is assumed that agents know the pricing function. It follows that the correct vector of states becomes

$$(\boldsymbol{x}_t^i)^{RE} = (S_{t-1}^i, B_{t-1}^i, \nu_t^i, \varepsilon_t^W, \varepsilon_t^D, \bar{\Gamma}_t)$$

Thus, knowledge of the current distribution and the expected future distributions is needed for agents to make optimal choices. Since the distribution is a highly dimensional object, this represents a severe complication of these models.

Instead, in our models we relax the information assumptions placed on the households' information set. We conjecture there is imperfect information about the distribution of wealth, income and expectations in the economy, so that despite agents being rational, they cannot use optimality conditions to derive equilibrium prices. Instead, they consider current prices as a state variable and forecast future prices directly using the subjective model of prices set up above, which is summarized by their current price growth expectations β_t^i. Thus, under internal rationality, the vector of states becomes

$$(\boldsymbol{x}_t^i)^{IR} = (S_{t-1}^i, B_{t-1}^i, \nu_t^i, \varepsilon_t^W, \varepsilon_t^D, P_t, \beta_t^i)$$

Equilibrium prices are determined by the stock market clearing condition:

$$\sum_i h^s(S_{t-1}^i, B_{t-1}^i, \nu_t^i, \varepsilon_t^W, \varepsilon_t^D, P_t, \beta_t^i) = 1 \tag{27}$$

Let Γ_t be the current cumulative distribution over asset holdings, wages and beliefs, that is,

$$\Gamma_t \equiv \Gamma(s, b, \nu, \beta) = \sum_{i=1}^{I} \mathbf{1}\Big\{ S_{t-1}^i \leq s, B_{t-1}^i \leq b, \nu_t^i \leq \nu, \beta_t^i \leq \beta \Big\} \tag{28}$$

When there is imperfect market knowledge, the equilibrium pricing function also depends on the distribution of beliefs, that is,

$$P_t = p(\Gamma_t, \varepsilon_t^W, \varepsilon_t^D) \tag{29}$$

The distribution Γ_t is needed to compute aggregate endogenous variables, but not for solving the households' problem.

Recursive Competitive Equilibrium. Given the exogenous processes, agents' probability measures, initial wealth holdings, and the international interest rate R, a recursive competitive equilibrium is given by

- The policy functions for consumption and investment share in risky assets:

$$\begin{pmatrix} C_t^i \\ v_t^i \end{pmatrix} = h^i((\boldsymbol{x}_t^i)^{IR}) \tag{30}$$

- The pricing function p:

$$P_t = p(\Gamma_t, D_t) \tag{31}$$

Such that

1. The first-order conditions, budget constraints and asset holdings limits of each

household are satisfied.

2. Markets clear.

Note that in the Rational Expectations version, one needs an additional function determining the law of motion of the distribution

$$\bar{\Gamma}_{t+1} = H(\bar{\Gamma}_t, \varepsilon_t^W, \varepsilon_t^D, \varepsilon_{t+1}^W, \varepsilon_{t+1}^D) \tag{32}$$

Households needs H to forecast the future distribution and then forecast future prices to make their current choices. Under Internal Rationality, though, this is not needed, as households use their subjective model of prices to directly forecast prices without any reference to current or future distributions. Under Internal Rationality, all it takes to simulate an equilibrium path is households' policy functions and the equilibrium pricing function, without explicit knowledge of H (given the exogenous processes, agents' probability measures, initial wealth holdings, and the international interest rate R). The distribution at the beginning of the period and the aggregate shocks pin down equilibrium prices. Equilibrium prices, aggregate shocks and predetermined variables are used to pin down allocations and update beliefs, which endogenously give rise to a new distribution.

Solving the Recursive Competitive Equilibrium. To solve the model, we need to find the functions h^c, h^v, p. We approximate these functions using parametric functions $\boldsymbol{\psi}$ of the states, following the idea of PEA. The choice of $\boldsymbol{\psi}$ is not obvious and not unique. Popular possibilities are polynomials, splines, neural networks, etc. Our approach is to try to find inspiration in economic theory, a bit following the idea of homotopy: take policy functions that are exact solutions for simpler models, and use a similar structure as a first guess.

For consumption, we conjecture the policy function is well approximated by

$$h^{c,i}(\boldsymbol{x}_t^i) \approx \psi_1(\boldsymbol{x}_t^i; \theta_1^i) = m_t^i Z_t^i \tag{33}$$

where θ is the vector of parameters in the approximating function,

$$m_t^i = 1 - \theta_1^i \beta_t^i \tag{34}$$

is the marginal propensity to consume out of wealth and

$$Z_t^i = W_t^i + (P_t + D_t)S_{t-1}^i + RB_{t-1}^i \tag{35}$$

Thus, the consumption function is linear on wealth, as in Benhabib, Cui, and Miao (2024). However, marginal propensities are idiosyncratic and time-varying, related to idiosyncratic beliefs about future returns.[7] Using ψ_1 in the budget constraint solves the consumption-savings problem. Yet, households still need to decide where to allocate savings. The key variable is the equity share v_t^i. We conjecture that households use the following rule to choice their portfolio composition

$$h^{v,i}(\boldsymbol{x}_t^i) \approx \psi_2(\boldsymbol{x}_t^i; \boldsymbol{\theta}_2^i) = max\Big\{\theta_{21}^i \theta_{22}^i + (1-\theta_{21}^i)\frac{\beta_t^i - R}{\sigma^i \gamma}, \underline{v}\Big\} \tag{36}$$

Households have a minimum level of desired stocks $\underline{v}$. Above it, they determine the equity share by using a linear combination between a fixed share θ_{22} and a share determined by their subjective Sharpe ratio. Thus, at the core of the problem there is the well-known result in Finance that a scaled Sharpe ratio determines the optimal portfolio choice (exactly in models with CARA preferences and Normal returns, approximately in models with CRRA and logNormal returns).

Using $h^{c,i}$ and $h^{v,i}$ in the budget constraint, the stock policy function is approximated by

$$h^{s,i}(\boldsymbol{x}_t^i) \approx (1-m_t^i)v_t^i \frac{Z_t^i}{P_t} \tag{37}$$

and for bonds,

$$h^{b,i}(\boldsymbol{x}_t^i) \approx (1-m_t^i)(1-v_t^i)Z_t^i \tag{38}$$

Using $h^{s,i}$ for all the agents in the stock market clearing condition and solving it for prices,

$$P_t = \frac{\sum_{i=1}^{1}(1-m_t^i)v_t^i\Big(W_t^i + D_t S_{t-1}^i + RB_{t-1}^i\Big)}{1 - \sum_{i=1}^{1}(1-m_t^i)v_t^i S_{t-1}^i} \tag{39}$$

which depends on the distribution of beliefs, wages and wealth at the beginning of the period. The particular guess of the policies allows us to compute equilibrium prices analytically, avoiding a complicated root-finding process. A benefit of this approach is that price determination is conceptually transparent and computationally fast.

[7] We simulate the model for the ratio of variables to dividends, but we keep here the non-normalized variables for the clarity of the exposition.

4. Quantitative analysis

Our quantitative exercise aims to measure the impact of heterogeneous expectations on wealth inequality. The calibration strategy follows three steps. First, we calibrate key distributional parameters using U.S. data, focusing on the wage distribution, aggregate income dynamics, and the empirically observed distribution of expectational fixed effects. Second, we ensure that the correlation between wages and expectations matches our empirical findings from the RAND panel. Then, we let the model generate its endogenous wealth distribution and evaluate its performance by comparing it with the data. Finally, to quantify the impact of heterogeneous expectations on wealth inequality, we simulate a counterfactual model where all households share identical beliefs calibrated to match the average expected returns in the data. By comparing this homogeneous-beliefs economy with our baseline model, we can isolate the contribution of expectational heterogeneity to wealth concentration.

Calibration Strategy

Table 5 presents our parameter choices. The aggregate parameters follow standard values in the macro-finance literature, with a quarterly discount factor of 0.99 and a risk aversion parameter of 2. The dividend process parameters are calibrated to match U.S. data, with mean growth of 2% and standard deviation of 1.9%. The wage-dividend ratio and its dynamics are chosen to match the capital income share and wage dynamics in the data.

Model Performance

Tables 6 and 7 assess the model's performance along various dimensions. The model generates aggregates which are reasonable. For instance, the equity premium is 8.57%, the capital income share of 25.79%, and a savings rate of 4.59%. Additionally, the safe-to-total assets ratio of 44.63% aligns well with aggregate portfolio allocations in the data.

Table 7 compares key distributional moments between our model and U.S. data. The model generates significant inequality in wealth, income, and returns across the wealth distribution, though it understates the degree of wealth concentration at the top. While the model predicts a top 1% wealth share of 10.64% compared to 38.5% in the data, it successfully captures the gradient in both portfolio returns and expectations. The top 1% achieves portfolio returns of 10.41% compared to 4.44%

Parameter	Symbol	Value	Source
	Panel A: Aggregate Parameters		
Discount factor	δ	0.99	Conventional
Mean dividend growth	β^D	2%	US data
Dividends growth standard deviation	σ_D	1.9%	US data
Risk-free rate	$R-1$	2.32%	US data
Risk aversion parameter	γ	2	Conventional
	Panel B: Distributional Parameters		
Correlation wages expectations	$\rho_{\bar{\beta}w}$	0.85	RAND panel
Pareto distribution parameter	α	2.5	Match wage distribution
Mean wage shares	μ	10	Match wage distribution
	Panel C: Expectation Parameters		
State persistence	ρ	0.90	RAND panel
Kalman gain	g	0.02	AMB(2017)
Fixed effects support	$[\underline{\beta},\ \bar{\beta}]$	[0.005,0.02]	Match expected returns

Table 5: Model Parameters.

		Model
Mean equity return	$\mathbb{E}(r_t^s)$	10.58%
Equity premium	$\mathbb{E}(r_t^s) - R - 1$	8.57%
Mean income growth	g^Y	2.11%
Mean capital income share	α_k	25.79%
Mean safe to total assets	$\mathbb{E}(B_t/(B_t + P_t S))$	44.63%
Mean savings rate	$\mathbb{E}(s_t)$	4.59%

Table 6: Model Performance: Aggregate Moments.

for the bottom 50%, close to the empirical spread of 8.30% to 3.40%. Similarly, expected returns exhibit a monotonic pattern across wealth groups, ranging from 8.63% for the top 1% to 4.41% for the bottom 50%, matching the empirical pattern.

The model also replicates key patterns in capital income. The capital income share increases sharply with wealth, with the top 1% earning 43.96% of their income from capital compared to 15.86% for the bottom 50%. While these numbers understate the empirical concentration (52.4% versus 3.8%), they capture the qualitative relationship between wealth and capital income. The wage distribution in the model aligns well with the data, particularly for the middle of the distribution, though it somewhat understates inequality at both extremes.

Table 8 shows that the model successfully replicates key features of expectation

By Wealth percentile	Model Top 1%	Model Top 10%	Model Middle 40%	Model Bottom 50%	Data Top 1%	Data Top 10%	Data Middle 40%	Data Bottom 50%
Wealth	10.64	49.80	37.41	12.79	38.5	77.1	21.7	1.2
Income	8.87	45.83	40.15	14.02	18.4	44.2	35	20.8
Wages	6.60	40.66	43.70	15.63	11.7	34	39.1	26.9
Capital income share	43.96	28.01	17.75	15.86	52.40	41.10	9.30	3.80
Portfolio return	10.41	7.24	4.95	4.44	8.30	6.12	3.63	3.40
Expected return	8.63	6.39	4.76	4.41	8.10	7.70	6.10	4.30

Table 7: Distributional Moments: Model vs Data. *This table compares distributional moments from the model with their empirical counterparts. All values are in percentages. Wealth, income, and wages show the share of total. Capital income share represents the fraction of total income from capital for each group. Portfolio and expected returns are average percentage returns for each group.*

formation documented in the survey data. The model generates significant extrapolation (b = 0.83) and overreaction (d = -0.30) coefficients, though it somewhat overstates the degree of extrapolation relative to the data. The model also matches the relative importance of individual fixed effects versus time effects in explaining expectation heterogeneity, with R-squared values of 0.54 and 0.04 respectively, close to their empirical counterparts of 0.50 and 0.01.

		Model	Data
Extrapolation coefficient	b	0.83***	0.07***
Overreaction coefficient	d	-0.30***	-0.37***
R^2 Individual Fixed Effects	$R^2_{\alpha^i}$	0.54	0.50
R^2 Time Fixed Effects	$R^2_{\mu_t}$	0.04	0.01
Correlation (expectations, wealth)	$\rho_{\beta,W}$	0.88	0.95

Table 8: Model Performance: Expectation Formation. **** indicates statistical significance at the 1% level.*

Figure 5 shows the joint distribution of wealth and expectations in the model. As in the data, an important fraction of the population is an area with low wealth and pessimistic beliefs and a tiny minority are wealthy and optimistic.

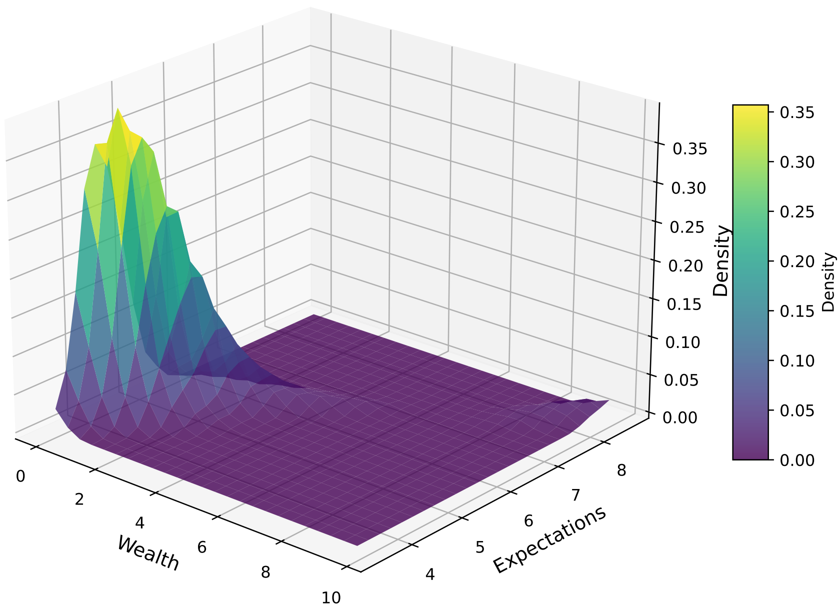

Figure 5: Joint distribution of wealth and expectations in the model.

The Impact of Heterogeneous Expectations on Wealth Inequality

Table 9 presents our key quantitative results, comparing distributional outcomes between our baseline model with heterogeneous expectations and a counterfactual economy where all households share identical beliefs. The contrast is striking: heterogeneous expectations substantially amplify wealth concentration. The wealth share of the top 1% increases from 7.02% under homogeneous expectations to 10.64% with belief heterogeneity, while the top 10% share rises from 40.71% to 49.80%. Figure 6 illustrates the effect. This amplification occurs through two channels: portfolio returns and capital income shares.

First, heterogeneous expectations generate substantial variation in portfolio returns across the wealth distribution. In the baseline model, the top 1% achieves returns of 10.41% compared to 4.44% for the bottom 50%. In contrast, returns are nearly identical across wealth groups under homogeneous expectations, hovering around 6.80%. Second, belief heterogeneity leads to dramatic differences in capital income shares. While these shares are uniform at roughly 26% under homogeneous expectations, they range from 43.96% for the top 1% to 15.86% for the bottom 50% in the baseline model.

Notably, consistent with Stachurski and Toda (2019), the wealth distribution

	Heterogeneous Expectations				Homogeneous Expectations			
By Wealth percentile	Top 1%	Top 10%	Middle 40%	Bottom 50%	Top 1%	Top 10%	Middle 40%	Bottom 50%
Wealth	10.64	49.80	37.41	12.79	7.02	40.71	43.66	15.62
Income	8.87	45.83	40.15	14.02	7.05	40.67	43.71	15.62
Wages	6.60	40.66	43.70	15.63	7.06	40.66	43.72	15.62
Capital income share	43.96	28.01	17.75	15.86	25.76	25.90	25.86	25.88
Portfolio return	10.41	7.24	4.95	4.44	6.80	6.79	6.80	6.79
Expected return	8.63	6.39	4.76	4.41	5.36	5.46	5.41	5.43

Table 9: The Effect of Heterogeneous Expectations. *This table compares distributional moments between our baseline model with heterogeneous expectations and a counterfactual economy with homogeneous expectations. All values are in percentages. The homogeneous expectations case uses the average expected return from the baseline model.*

under homogeneous beliefs inherits the tail behavior of the income distribution under homogeneous expectations. However, heterogeneous expectations generate additional skewness through their effects on portfolio choices and savings behavior, substantially amplifying wealth concentration at the top of the distribution.

A Microfoundation for Heterogeneous Beliefs: Learning from experience

While our baseline model successfully captures the joint distribution of expectations and wealth, it takes a reduced-form approach by directly imposing fixed effects in beliefs and their correlation with wages. We now develop a microfoundation for these patterns based on learning from experience. The key insight is that initial differences in portfolio choices lead to heterogeneous realized returns, which in turn generate persistent differences in beliefs through learning if agents use their own portfolio returns as signal for aggregate returns.

To capture that idea, we model returns as having both a common and an idiosyncratic component:

$$\begin{aligned} lnR_t &= \mu lnb_t^i + (1-\mu)lnR_t^i + ln\varepsilon_t^{R,i} \\ lnb_t^i &= lnb_{t-1}^i + ln\varepsilon_t^{b,i} \end{aligned} \tag{40}$$

where R_t^i represents household i's portfolio return and b_t^i captures persistent individual effects. Households learn about returns through a modified Kalman filter:

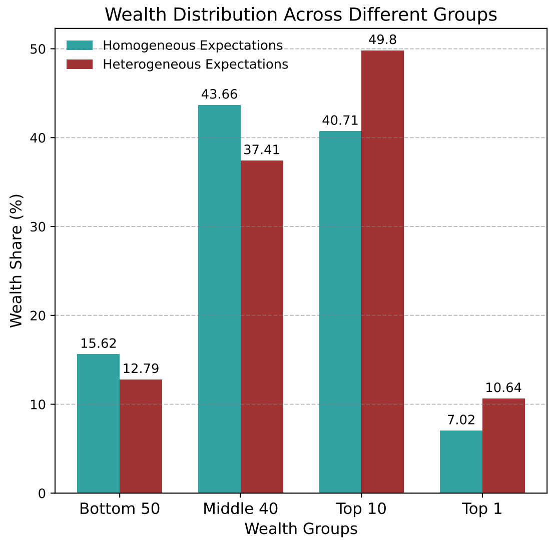

Figure 6: Wealth distribution with and without heterogeneous expectations.

	Het. Risk Aversion				+ Learning about own return			
By Wealth percentile	Top 1%	Top 10%	Middle 40%	Bottom 50%	Top 1%	Top 10%	Middle 40%	Bottom 50%
Wealth	9.34	45.96	39.93	14.11	14.36	52.47	35.38	12.15
Capital income share	48.85	39.55	30.54	28.45	83.01	34.62	21.14	18.46
Portfolio return	5.13	4.03	3.12	2.95	8.84	4.72	3.31	2.96
Expected return	9.70	9.69	9.49	9.25	9.06	5.40	4.09	3.79

Table 10: The Amplification Effect of Learning from Experience. *This table compares distributional outcomes between an economy with only heterogeneous risk aversion and one that adds learning from portfolio returns. All values are in percentages.*

$$ln\beta_t^i = ln\beta_{t-1}^i + g^i\Big(lnR_{t-1} - lnR_{t-1}^i + \mu(R_t^i - \beta_t^i)\Big) + g^i\varepsilon_t^{R1,i} \tag{41}$$

This learning process generates expected returns that depend on both beliefs about the common component and individual portfolio performance:

$$\mathbb{E}_t^{\mathcal{P}^i}(R_{t+1}) = \kappa^i(\beta_t^i)^\mu(R_t^i)^{1-\mu} \tag{42}$$

Table 10 demonstrates how learning from experience amplifies wealth inequality. We compare two economies: one with only heterogeneous risk aversion and another that adds learning from experience. The learning mechanism substantially increases wealth concentration: the top 1% wealth share rises from 9.34% to 14.36%, while their capital income share jumps from 48.85% to 83.01%.

Crucially, learning generates a feedback loop that amplifies initial differences: heterogeneous portfolios lead to different realized returns, which through learning create heterogeneous expectations, further reinforcing portfolio heterogeneity. This mechanism transforms initially similar expectations (varying only from 9.25% to 9.70% across wealth groups under pure risk aversion heterogeneity) into highly dispersed beliefs (ranging from 3.79% to 9.06% with learning).

These results suggest that learning from experience can provide a unified explanation for the joint distribution of returns, expectations, and wealth observed in the data. Moreover, it identifies a novel amplification mechanism: the interaction between portfolio heterogeneity and belief formation through learning can substantially magnify wealth inequality beyond what standard theories would predict.

5. Conclusion

This paper documents substantial heterogeneity in households' return expectations using survey data. We find persistent disagreement about future returns that defies the common "communism of beliefs" assumption in macroeconomics. Importantly, optimism about returns shows a strong positive correlation with both income and wealth, suggesting a potential amplification mechanism for wealth inequality.

We develop a model where households learn about market price data while maintaining heterogeneous long-run views about the fundamental value of the asset. The model successfully replicates key features of the micro data, including the persistence of belief heterogeneity and its correlation with wealth. Our approach based on Internal Rationality not only makes the model consistent with survey evidence but also substantially simplifies the solution of heterogeneous agent models with aggregate risk, addressing recent critiques about their computational tractability.

Quantitatively, we find that heterogeneous expectations significantly amplify wealth inequality. When we shut down belief heterogeneity in our calibrated model, the wealth share of the top 1% falls by about 50%, and the dispersion in portfolio returns nearly disappears. These results suggest that accounting for heterogeneous expectations is crucial for understanding the dynamics of wealth concentration.

Looking ahead, our framework opens several promising research directions. First, the simplified solution method we develop could be applied to study other sources of heterogeneity in macro models with aggregate risk. Second, our learning mechanism could be extended to understand the transmission of beliefs across generations or through social networks. Finally, the strong link we document between expectations and wealth calls for further investigation of policy interventions that might affect this relationship, such as financial education or information provision. If beliefs, though, are too rooted to change, alternative options such as sovereign wealth funds might be a good strategy to socialize risk and return.

These findings have important implications for both research and policy. For researchers, they highlight the importance of incorporating realistic expectation formation in macro models. For policymakers, they suggest that policies affecting belief formation and financial education might have significant distributional consequences through their impact on portfolio choices and wealth accumulation.

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